Working Paper: Commercial Real Estate Mortgage Default Accounting for Utility Expenditures

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Abstract

This report presents empirical results from the estimation of reduced form models of commercial mortgage default when accounting for the utility expenditures of individual commercial buildings. Our empirical specification accounts both for proxies related to traditional default-option exercise factors, such as the likelihood of negative equity, proxied by the origination loan-to-value ratio of the loan, and percentage of utility costs to total revenues (expenses). We find that both the option exercise channel and the energy-cost characteristics of the buildings are importantly associated with mortgage default for securitized multifamily, retail, and office loans. These results provide a novel extension to the current literature on "double trigger" controls for default, by showing that the effect of shocks to energy factor inputs is statistically related to commercial mortgage distress. We find that an historically likely 30% shock to energy prices leads to a 30 basis points increase in multifamily mortgage default, a 90 basis points increase in retail mortgage default, and a 50 basis points increase in office mortgage default. These estimates indicate that our overall results are not just statistically significant but also that the ratio of utility costs to total expenses has an economically relevant effect on the incidence of commercial mortgage default.

Key words: Mortgages, Energy risk.

JEL codes: G21

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1 Introduction

Prior studies have found that commercial mortgage defaults are associated with various measures of the pricing, consumption, and volatility of building energy use (see, Issler, Mathew, Sun, and Wallace, 2017; Issler, Mathew, and Wallace, 2020a,b, 2021). Other studies have shown that energy ratings, such as LEED certification and ENERGY STAR, as well as the wholesale market prices and volatility of electricity and natural gas can affect commercial building values and mortgage default (see, for example, Eichholtz, Kok, and Quigley, 2010; Issler, Jaffee, Stanton, and Wallace, 2019; Jaffee, Stanton, and Wallace, 2018). Our study contributes to this literature in two ways. First, we introduce a new measure of utility costs based on building-level financial statements, sourced from the Commercial Real Estate Financial Council (CREFC), to evaluate the energy consumption and efficiency of commercial properties. Second, the new CREFC financial statement data are merged to the loan-level TREPP commercial mortgage origination and performance data, thus allowing for analyses of building-specific energy efficiency on the default performance of securitized retail, multi-family, and commercial mortgages for the entire U.S.

Our default analysis follows standard methods to estimate reduced-form mortgage default based on the likelihood that the borrower will experience negative equity, where the loan amount (or its market price) exceeds the value of the building. (see, for example, Ciochetti, Deng, Lee, Shilling, and Yao, 2003; Clapp, An, and Deng, 2006; Deng, Quigley, and Van Order, 2000; Schwartz and Torous, 1989; Stanton and Wallace, 2018; Titman and Torous, 1989). Our proxy for this likelihood is the origination loan-to-value ratio, where a higher initial loan-to-value ratios implies a greater potential exposure to negative equity events in the future.

We also draw upon the more recent "double trigger" reduced form models of commercial default, in which it is not just the degree of expected negative equity of the borrower's debt position that triggers default, but also shocks associated with loss of cash flow that stress the borrowers ability or willingness to make the debt service payments, due to loss of tenants, increases in operating costs, or both. (see, for example, An, Fisher, and Anthony, 2015; An and Sanders, 2010; Capone and Golding, 2002; Cunningham, Gerardi, and Shen, 2021; Issler et al., 2021; Riddiough and Thompson, 1993). Our focus is on the importance of utility costs, including water, natural gas, and electricity, as a percentage of total operating revenues measured at the end of the last observed operating year. In our data, we find that on average annualized utility costs account for 15% (standard deviation of 6%) of total operating expenses for retail properties, 20% (standard deviation of 8%) of total operating expenses for office properties, and 17% (standard deviation of 5%) of total operating expenses for multifamily buildings. Our estimation strategy thus controls for both the negative equity channel to default (the loan-to-value ratio) and the triggering effects of energy-cost related cash flow shocks (level of utility costs to revenues) on the mortgage default risk of commercial real estate loans.

This paper is organized as follows. We discuss the TREPP-CREFC data in Section 3. Section 4 presents the summary statistics for the data set. The setup for the estimation of the linear default model presented is Section 5 and Section 7 concludes.

2 Measurement of property-specific energy efficiency

Our previous studies of commercial mortgage defaults applied a number of different measures of energy efficiency and energy consumption. Wallace, Issler, Mathew, and Sun (2017), employed Source Energy Use Intensity (Source EUI) as the building measure of energy efficiency.[1] Issler et al. (2020a,b) used Scaled Source EUI, a measure of Source EUI that is scaled to the Net Operating Income (NOI) of the property per square foot, as the primary measure of energy efficiency. Their motivation for scaling Source EUI by NOI per square foot was that this new variable was better able to capture the magnitude of the contribution of energy costs to the borrower's ability to serve the property mortgage. The higher the EUI (energy use per square foot), the higher the energy costs per square foot, as compared to other more efficient buildings. And the higher the energy costs relative to NOI (both scaled by square foot), the lower the ability of the borrower to pay its mortgage dues (see Issler et al., 2020a). Issler et al. (2021) used the Fannie Mae DUS data on the utility costs and net cash flows for multifamily properties and again constructed a scaled version of the measure as the total utility expenditures of the building divided by the net cash flow, assumed to be a reasonable approximation of net operating income.

The availability of energy-related cost and mortgage origination and performance data at the building level has been a major impediment to research focused on the role of energy efficiency on debt performance. An especially important missing link has been the availability of vetted financial information, such as the total annual dollar expenditures on utilities with comparable vetted data for the total revenues of buildings. One effort to address this gap in needed cost information were prior DOE-funded studies that introduced a location-based measure of building-specific electricity price risks (see Issler et al., 2020a,b, 2021). This measure, termed as the electricity price gap, was constructed as the difference between the

[1] See, https://www.energystar.gov/buildings/facility-owners-and-managers/existing-buildings/use-portfolio-manager/understand-metrics/difference. Source EUI is considered the "gold standard" measure of energy efficiency because it provides the most equitable assessment of building-level energy efficiency.

forecasted and actual electricity wholesale prices for a building over the mortgage holding period. It is a proxy for the total energy expenditures, for which we had no direct measurement. As a result, we could not directly relate actual building-level energy expenditures to the performance of their loans (see Issler et al., 2020a,b, 2021).

All of these studies were hampered by small sample sizes due to the challenges of merging building-level energy efficiency data with data on the origination and performance statistics for mortgages for the same buildings (see, Issler et al., 2017, 2020a,b). Thus, inference to the larger population of commercial mortgages in the U.S. was compromised by potentially non-representative convenience samples. A limitation with the Fannie Mae DUS data was that the reported utility data was not available for the preponderance of the defaulted multifamily loans, so that the defaulted loan sample was limited to only a few states such as Texas, Oklahoma, and Colorado (see Issler et al., 2021).

Finally, prior to Fall of 2021, the TREPP data, which is the most trusted source for securitized commercial mortgage origination and performance data in the U.S., provided no information on the financial statements of the buildings including crucial information on quarterly or annual utility costs and comparable information on revenues and operating expenditures. However, as a result of the merge of the CREFC and the TREPP data, we can now take advantage of a large national study of energy efficiency and mortgage default based upon a sample of more than twenty eight thousand fixed rate commercial loans.

3 TREPP Commercial Mortgage and Commercial Real Estate Finance Council's (CREFC) Utility Data

We estimate the default probability models in this report using a sample of 28,224 commercial loans originated between 2000 and 2021. These data were obtained from Trepp LLC loan-level origination and performance data and include information on the structure of the mortgage contract, the property and leasehold characteristics, and monthly performance records for each mortgage. TREPP obtains its loan-level origination and performance information from master servicer reports using a standardized reporting format defined by the Commercial Real Estate Finance Council's Investor Reporter Package (IRP). The TREPP data reports monthly observations of loan's performance including the status of the loan, such as prepaid, delinquent, foreclosed or current in each month. It also contains updated occupancy rates and loss information, if reported by the servicer, as well as detailed information on the contractual structure of the loan at origination. In our study, we considered loans that were originated for single properties so each individual loan can be tied to a specific property and its operating performance. We also focus on fixed rate mortgages that can be either fully amortizing, partially amortizing, or interest only loans.

As discussed above, the utility data for the study were obtained from the Property Financial Data set (PFD) compiled by the Commercial Real Estate Finance Council (CREFC) and merged with the TREPP master data sets. CREFC is the trade association for the commercial real estate finance industry. Its member firms include balance sheet and securitized lenders, loan and bond investors, private equity firms, servicers and rating agencies, and others. CREFC's role in the commercial finance industry is to establish market standards for reporting commercial real estate financial information at the loan level in support of the operation of the securitized commercial mortgage market and its bond investors. The PFD includes detailed operating income and expense data for the buildings that are the collateral for each loan. CREFC reports these data in total dollar values per year and the standardizes the data by square footage for office and retail buildings and by unit count for multifamily buildings. Importantly, CREFC breaks down the operating income and expense data and reports the total utility costs in dollars.

4 Summary Statistics

Table 1 tabulates the mean and standard deviations for the key variables of our study. The loans that are included in the sample are either i) current, defined by non-defaulted loans that are still outstanding or matured in the period between 2002 and 2021), or ii) defaulted, characterized by 60 days or more of delinquency, or a more severe event such as foreclosure, REO or bankruptcy.[2] Out of 18,832 multifamily observations, 243 loans (1.29%) entered into a defaulted state; out of 5,128 retail loans, 396 loans (7.72%) entered into a defaulted state, and lastly of 4,264 office loans, 291 (6.82%) entered into a defaulted state. We measure the utility cost as a percentage of total revenues, or total expenses, and the loan age on the last observation day of the loan, which again is its maturity date, its last reported performance month, or the month of its default. As shown, the office properties have the highest mean

[2] At this time were are not considering loans that prepaid or defeased. Defeasance is a form of prepayment penalty that requires the borrower to replace the remaining loan balance with a portfolio of treasury securities that exactly replicate the remaining owed installment payments on the loan.

Table 1: Overall Summary Statistics for the Covariates

	Mean	Std. Dev.	Minimum	Maximum
Multifamily loans (N = 18,832)				
Last Observation Utility Cost to Total Revenues (%)	7.89	3.35	2.01	24.92
Last Observation Utility Cost to Total Expenses (%)	17.04	5.27	8.00	30.99
Last Observation Loan Age (months)	89.10	58.80	2.00	244.00
Origination Loan to Value Ratio (%)	68.15	10.87	5.50	80.00
Origination Mortgage interest rate (%)	4.83	.94	1.68	8.93
Origination Amortization period (months)	342.22	93.65	18	427.00
Origination Balance due date (months)	111.33	33,21	18.00	420.00
Origination Balance ($ Millions)	13.88	17.12	6.00	268.74
Interest Only Loan Indicator	56.00%		0.00	1.00
Default Indicator	1.29%		0.00	1.00
Retail loans (N = 5,128)				
Last Observation Utility Cost to Total Revenues (%)	5.35	2.95	2.01	24.84
Last Observation Utility Cost to Total Expenses (%)	15.00	5.60	7.00	29.98
Last Observation Loan Age (months)	151.43	52.68	5.00	245.00
Origination Loan to Value Ratio (%)	68.32	10.12	17.7	80.00
Origination Mortgage interest rate (%)	5.58	0.74	2.72	8.75
Last Observation Loan Age (months)	151.43	52.68	5.00	245.00
Origination Amortization period (months)	333.75	89.21	24.00	425.00
Origination Balance due date (months)	119.96	22.42	24.00	300.00
Origination Balance ($ Millions)	10.07	13.17	.15	210.00
Interest Only Loan Indicator	45.00%		0.00	1.00
Default Indicator	7.72%		0.00	1.00
Office loans (N = 4,264)				
Last Observation Utility Cost to Total Revenues (%)	10.28	4.50	2.02	24.95
Last Observation Utility Cost to Total Expenses (%)	22.17	7.56	9.01	40.97
Last Observation Loan Age (Months)	144.93	57.92	1.00	244.00
Origination Loan to Value Ratio (%)	67.99	10.72	4.20	80.0
Origination Mortgage interest rate (%)	5.55	0.83	2.46	10.99
Origination Amortization period (months)	318.40	108.45	24.00	428.00
Origination Balance due date (months)	114.18	22.58	24.00	300.00
Origination Balance ($ Millions)	20.99	33.25	.33	1,125.00
Interest Only Loan Indicator	54.15%		0.00	1.00
Default Indicator	6.83%		0.00	1.00

utility cost as a percentage of total revenues at 10.28% (standard deviation of 4.5%), multifamily loans have the next highest at 7.89% (standard deviation of 3.35%), and retail properties have the lowest percentage at 5.35% (standard deviation (2.95%). Similarly, the office properties have the highest mean utility cost as a percentage of total expenses at 22.17% (standard deviation of 47.56%), multifamily loans have the next highest at 17.04% (standard deviation of 5.27%), and retail properties have the lowest percentage at 15.00% (standard deviation (5.60%).

The retail and office loans have roughly equivalent loan ages on the last observation month, with an average of 154 months for the retail loans and 145 months for the office loans, whereas the multifamily loans average is only 89 months outstanding. The average amortization maturity terms are very similar across the three property types as are the average due dates for the payment of the mortgage balances. The average amortization periods are very similar across the three property types as are the the average due dates for the payment of the mortgage balances. The average origination balance of the office loans is $20.99 million whereas the average retail loan size is $10.07 million and the multifamily loans average is $13.88 million. Other important similarities among the loan types include the average origination loan-to-value ratios and the origination interest rate on the loans. Finally, the multifamily and office loan data have a higher percentage of loans that are interest only (i.e., make no payments to pay down principal over the maturity period) than the retail loans. Usually, loans that do not amortize are considered riskier because of their higher outstanding balance at the time of refinancing, defined by the loan's balloon date or maturity date.

Table 2 tabulates the mean and standard deviations for the defaulted and non-defaulted samples. As shown, the average utility cost as a percentage of total revenues and the average utility cost as a percentage of total expenses are both uniformly higher for the defaulted than for the non-defaulted loans. Additionally, all of the defaulted loans are on average older, especially for the multifamily loans. The defaulted multifamily and office loans are on average collateralized by less valuable buildings, whereas the the appraised value of the defaulted retail loans are slightly larger. Other important summary statistic distinctions between the defaulted and non-defaulted loans are the consistently higher average percentage of interest-only loans among the defaulted loans, the consistently higher average origination loan-to-value ratios of the defaulted loans, and the consistently higher origination interest rate of the defaulted loans. All those factors reflect the higher origination risk of the loans.

Overall, the summary statistics indicate that there are important average contractual and performance differences among the loans by property type and more importantly by the defaulted and non-defaulted loans. Moreover, the differences between the percentage of utility costs to total revenues for the defaulted and non-defaulted loans on the last observation day is very suggestive that utility costs may be a key stress factor for the ability, or willingness, of the borrower to make mortgage payments given the property revenues. This

Table 2: Selected Summary Statistics for the Defaulted Versus Non-Defaulted Loans

	Mean	Std. Dev.	Mean	Std. Dev.
	Non-Defaulted		Defaulted	
Multifamily loans	N = 18,589		N=243	
Utility Cost (% of Total Expenses)	17.02	5.268	18.83	5.50
Utility Cost (% of Total Revenues)	7.85	3.31	11.20	4.60
Last Observation Age of Loan (months)	88.24	58.38	153.96	53.84
Secured Appraised Value of Building ($ Millions)	25.890	32.25	15.367	14.791
Interest Only Loan (%)	55.82%		70.73%	
Origination Loan-to-Value Ratio (%)	68.06	10.89	74.96	6.01
Origination Interest Rate (%)	4.82	0.93	5.67	0.75
Retail loans	N = 4,739		N=396	
Utility Cost (% of Total Expenses)	14.93	5.578	15.76	5.75
Utility Cost (% of Total Revenues)	5.20	2.81	7.10	3.90
Last Observation Age of Loan (Months)	150.05	53.80	167.96	32.43
Secured Appraised Value of Building ($ Millions)	17.49	32.48	18.69	26.41
Interest Only Loan (%)	43.34		68.43	
Origination Loan-to-Value Ratio	67.96	10.25	72.77	6.98
Origination Interest Rate	5.57	0.75	5.74	0.53
Office loans	N = 3,973		N=291	
Utility Cost (% of Total Expenses)	22.05	7.52	23.83	7.83
Utility Cost (% of Total Revenues)	10.06	4.35	13.25	5.39
Last Observation Age of Loan (Months)	142.91	58.95	172.33	29.94
Secured Appraised Value of Building ($ Millions)	42.28	89.48	25.45	35.62
Interest Only Loan (%)	52.44		77.40	
Origination Loan-to-Value Ratio	67.61	10.88	73.09	6.67
Origination Interest Rate	5.53	0.84	5.83	0.95

channel–in addition to the option-related negative equity channel, measured by the origination loan-to-value ratios, that are also significantly different between the defaulted and non-defaulted loans–will be explored in the next section using linear probability and logistic regression models for measuring the probability of default. Finally, another feature that is not reported in the summary statistics is the geographic distribution of the loans, with more concentration in the states of Texas, California, Florida, New York and Massachusetts, in that order.

5 Estimation Results

We estimate two different models for the probability of commercial mortgage default outcomes. The first is a linear probability model that assumes that the error structure is normally distributed. In this setting one cannot guarantee that the predicted outcomes are strictly between zero and 1 as probabilities should be. The second specification accounts for the fact that the left hand side variable can only vary between zero and one and therefore applies a linearization of a logistic function form (i.e., the left hand side variable is the natural log of the odds ratio between default and non-default). Interestingly, the results of the two specifications are remarkably similar, suggesting that the covariates are well balanced between default and non-default (i.e., the 0 and 1 realizations). We consider two measures for the effect of utilities on default: 1) utilities measured as percentage of total revenues; 2) utilities measured as percentage of total expenses.

5.1 Utilities measured as percentage of total revenues

Table 3 presents the results of the linear probability model. The results for the multifamily loans are reported in the upper panel of the table. As shown, the largest positive and statistically significant effects on the default outcome are the utility cost as a percentage of revenue on the last observation month, the origination interest rate of the loan, and an indicator variable for whether the loan was structured as an interest only loan. All these covariates are associated with the borrower's willingness or ability to cover the remaining installment payments or the remaining balance on the loan. The higher these costs the higher the likelihood there will be a default on the loan. The next largest and statistically positive effect is the proxy for the likelihood of negative equity on the property as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the loan balance is statistically significant at the .001 level. The percent of variance explained by the overall regression, measured by its R^2, is only 3.65%; however, the default incidence in the multifamily loans was about 1% over the analysis period ending in 2021.

The results for the retail loans are presented in the middle panel of Table 3. Here again, the three strongest positive and statistically significant effects on the probability of default are the utility cost as a percentage of revenue on the last observation month, the interest rate on the loan (although its statistical power is somewhat less strong at 5%), and an indicator variable for the whether the loan was structured as an interest only loan. As above, all these covariates are associated with the willingness or ability to pay the remaining installments or remaining balance on the loan, whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the origination amortization term and the balance due term are significant at the .001 level. The percent of variance explained by the overall regression, measured by its R^2, is 7.8%; however, the default incidence in the retail loans was about 7.72% over the analysis period ending in 2021.

Finally, the results for the office loans are presented in the lower panel of Table 3. The same pattern is repeated here again, with the three strongest positive and statistically significant effects on the probability of default being the utility cost as a percentage of revenue on the last observation month, the interest rate on the loan (although its statistical power is somewhat less strong at 5%), and the indicator variable for the whether the loan was structured as an interest-only loan. As above, all of these covariates are associated with the willingness or ability to cover the remaining installment payments, whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the the origination amortization term and the balance due term are statistically significant at the .001 level. The percent of variance explained by the overall regression (the R^2) is only 7.38%; however, the default incidence in the office loans was only about 6.83% over the analysis period ending in 2021.

Table 4 presents the results of the logistic regression model for the multifamily loans in the upper panel of the table. Exactly as with the prior results, the largest positive and statistically significant effects (with χ^2 tests that reject the null at $< .0001$) on the default outcome are the utility cost as a percentage of revenue on the last observation month, the origination interest rate on the loan, and the indicator variable for the whether the loan was structured as an interest-only loan. Again, all of these covariates are associated with the willingness or ability to cover the remaining installment payments or the remaining balance which is the present value of these payments on the loan. The higher these costs, the greater the likelihood there will be a default. The next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the origination balance and the origination amortization term are statistically significant at the .001 level. The likelihood ratio test indicates that the null of zero value coefficients is rejected at $< .0001$.

The results for the retail loans are presented in the middle panel of Table 4. Like the linear

Table 3: **Linear probability estimates of default for multifamily, retail, and office mortgages where utilities are measured as percentage of total revenues.** This table presents the coefficient estimates for a linear regression of the probability of default (measured as a zero or one indicator variable) based upon samples of commercial mortgages by property type that are evaluated at the last observable month of performance for current loans and on the last recorded performance month for defaulted loans. We control for the origination characteristics of the mortgage, the age of the loan on the last observation day, state fixed effects and two default related channels: i) the likelihood of negative equity (measured as the origination loan-to-value ratio); ii) a measure of energy-related cost effects on cash flow on the last observation day, the "double trigger," (measured as the total cost of utilities as a percentage of total revenues).

Multifamily	Coefficient Estimate	Standard Error
Intercept	-.083***	0.009
Utility Cost to Total Revenues (%)	0.0035***	0.0003
Age of loan (Months)	0.00016***	0.000019
Origination Balance ($ Millions)	0.00014**	0.00005
Origination Loan-to-value ratio	0.0005***	0.00008
Origination Mortgage Interest rate	0.0087***	0.00125
Origination Amortization Term (Months)	-0.00004***	0.000009
Origination Balance Due Term (Months)	-0.00017***	0.000027
Interest Only Loan Indicator	0.0139***	0.00185
State fixed effects	Yes	
Number of observations	18,832	
R^2	0.0365	
Retail	Coefficient Estimate	Standard Error
Intercept	-.354***	0.043
Utility Cost to Total Revenues (%)	0.0157***	0.0012
Age of loan (Months)	0.00038***	0.000082
Origination Balance ($ Millions)	0.0015***	0.00029
Origination Loan-to-value ratio (%)	0.002***	0.00039
Origination Mortgage Interest rate (%)	0.0163**	0.00602
Origination Amortization Term (Months)	-0.000099	0.0004
Origination Balance Due Term (Months)	-0.000057	0.00016
Interest Only Loan Indicator	0.0769***	0.00767
State fixed effects	Yes	
Number of Observations	5,128	
R^2	0.078	
Office	Coefficient Estimate	Standard Error
Intercept	-0.2603***	0.04240
Utility Cost to Total Revenues (%)	0.0085***	0.00088
Age of loan (Months)	0.00035***	0.00008
Origination Balance ($ Millions)	0.00007	0.00012
Origination Loan-to-value ratio (%)	0.0018***	0.00038
Origination Mortgage Interest rate (%)	0.0135**	0.0058
Origination Amortization Term (Months)	-0.00004	0.000039
Origination Balance Due Term (Months)	-0.000015	0.000017
Interest Only Loan Indicator	0.0772***	0.00816
State fixed effects	Yes	
Number of Observations	4,264	
R^2	.0738	

$^{***}P < 0.01$, $^{**}P < 0.0$

Table 4: **Logistic regression estimates of the probability of default for multifamily, retail, and office mortgages where utilities are measured as percentage of total revenues.** This table presents the coefficient estimates for a logistic regression of the probability of default (measured as a zero or one indicator variable) based upon samples of commercial mortgages by property type that are evaluated at the last observable month of performance for current loans and on the last recorded performance month for defaulted loans. We control for the origination characteristics of the mortgage, the age of the loan on the last observation day, state fixed effects and two default related channels: i) the likelihood of negative equity (measured as the origination loan-to-value ratio); ii) a measure of energy-related cost effects on cash flow on the last observation day, the "double trigger," (measured as the total cost of utilities as a percentage of total revenues).

Multifamily	Coefficient Estimate	Wald Error	χ^2	Prob. $> \chi^2$
Intercept	-14.3653	1.0343	192.8947	$< .0001$
Utilities as % Revenues	0.1651	0.0157	111.0645	$< .0001$
Age of Loan (Months)	0.0103	0.00166	38.4148	$< .0001$
Origination Balance ($ Millions)	0.00733	0.00636	1.3278	0.2492
Origination Loan-to-Value Ratio (%)	0.0685	0.0119	33.4225	$< .0001$
Origination Interest Rate (%)	0.6954	0.1031	45.5118	$< .0001$
Origination Amortization Term (Months)	-.00123	0.000690	3.1757	.0747
Origination Balance Due Term (Months)	-0.0149	0.00283	27.7647	$< .0001$
Interest Only Loan Indicator	1.1154	0.1628	46.9485	$< .0001$
State Fixed Effects	Yes			
Number of observations	18,832			
Likelihood Ratio = 612.2393	Df = 13		Prob. $\chi^2 < .0001$	
Retail	Coefficient Estimate	Wald Error	χ^2	Prob. $> \chi^2$
Intercept	-9.9098	0.8265	143.7649	$< .0001$
Utilities as % Revenues	0.1555	0.0141	122.36745	$< .0001$
Age of Loan (Months)	0.00704	0.00159	19.5134	.0638
Origination Balance ($ Millions)	0.0158	0.00337	21.9316	0.2492
Origination Loan-to-Value Ratio (%)	0.0443	0.00776	32.5553	$< .0001$
Origination Interest Rate (%)	0.2894	0.00159	19,5134	$< .0001$
Origination Amortization Term (Months)	-.00122	0.000690	3.1757	.0747
Origination Balance Due Term (Months)	-0.00369	0.00316	1.3662	.2425
Interest Only Loan Indicator	1.1304	0.1220	85.8824	$< .0001$
State Fixed Effects	Yes			
Number of observations	5,128			
Likelihood Ratio = 325.1955	Df = 13		Prob. $\chi^2 < .0001$	
Office	Coefficient Estimate	Wald Error	χ^2	Prob. $> \chi^2$
Intercept	-10.9432	0.9959	120.7495	$< .0001$
Utilities as % Revenues	0.1159	0.0132	76.8427	$< .0001$
Last Observation Age of Loan (Months)	0.0101	0.00209	23.3370	$< .0001$
Origination Balance ($ Millions)	-0.00322	0.00316	1.0376	0.3084
Origination Loan-to-Value Ratio (%)	0.0522	0.00967	29.1675	$< .0001$
Origination Interest Rate (%)	0.2912	0.1094	7.0803	.0078
Origination Amortization Term (Months)	-.00061	0.000643	.9138	.3391
Origination Balance Due Term (Months)	-0.00357	0.00323	1.2201	.2693
Interest Only Loan Indicator	1.3381	0.1562	73.3425	$< .0001$
State Fixed Effects	Yes			
Number of observations	4,264			
Likelihood Ratio = 359.8220	Df = 13		Prob. $\chi^2 < .0001$	

regression model, again the three strongest positive and statistically significant effects on the probability of default are the utility cost as a percentage of revenue on the last observation month, the interest rate on the loan, and the indicator variable for the whether the loan was structured as an interest-only loan. As above, all of these covariates are associated with the willingness or ability to pay the remaining installment payments on the loan whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the original loan balance, the origination amortization term and the balance due term are significant at the .001 level. The likelihood ratio test indicates that the null of zero value coefficients is rejected at $< .0001$.

Finally, the results for the office loans are presented in the lower panel of Table 3. Here again, the same pattern is repeated with the three strongest positive and statistically significant effects on the probability of default being the utility cost as a percentage of revenue on the last observation month, the interest rate on the loan (although its statistical power is somewhat less strong at .01%), and the indicator variable for the whether the loan was structured as an interest-only loan. As above, all of these covariates are associated with the willingness or ability to cover the remaining installment payments on the loan whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the original loan balance, the origination amortization term and the balance due term are significant at the .001 level. The likelihood ratio test indicates that the null of zero value coefficients is rejected at $< .0001$.

In terms of elasticities, holding everything else constant, the linear probability model indicates that a 1% increase in the annualized utility costs as a percentage of total revenues increases default risk by 35 basis points for multifamily loans, by 157 basis points for retail loans, and by 85 basis points for office loans. The elasticity of negative equity for a 1% change in the loan-to-value ratio are 5 basis points for multifamily loans, 20 basis points for retail loans, and 18 basis points for office loans.

5.2 Utilities measured as percentage of total expenses

Table 5 presents the results of the linear probability model. The results for the multifamily loans are reported in the upper panel of the table. As shown, the largest positive and statistically significant effects on the default outcome are the utility cost as a percentage of expenses on the last observation month, the origination interest rate of the loan, and an indicator variable for whether the loan was structured as an interest-only loan. All these covariates are associated with the borrower's willingness or ability to cover the remaining installment payments or the remaining balance on the loan. The higher these costs, the higher the likelihood there will be a default on the loan. The next largest and statistically positive effect is the proxy for the likelihood of negative equity on the property as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the loan balance is statistically significant at the .001 level. The percent of variance explained by the overall regression, measured by its R^2, is only 2.90%; however, as discussed above the default incidence in the multifamily loans is about 1% over the analysis period ending in 2021.

The results for the retail loans are presented in the middle panel of Table 5. Here again, the three strongest positive and statistically significant effects on the probability of default are the utility cost as a percentage of expenses on the last observation month, the interest rate on the loan (although its statistical power is somewhat less strong at 5%), and an indicator variable for the whether the loan was structured as an interest-only loan. As above, all these covariates are associated with the willingness or ability to pay the remaining installments or remaining balance on the loan, whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the the origination amortization term and the balance due term are significant at the .001 level. The percent of variance explained by the overall regression, measured by its R^2, is 5.1%; however, as discussed above the default incidence in the retail loans was about 7.72% over the analysis period ending in 2021.

Finally, the results for the office loans are presented in the lower panel of Table 3. Here again, the same pattern is repeated with the three strongest positive and statistically effects on the probability of default being the utility cost as a percentage of expenses on the last observation month, the interest rate on the loan (although its statistical power is somewhat less strong at 5%), and the indicator variable for the whether the loan was structured as an interest-only loan. As above, all of these covariates are associated with the willingness or ability to cover the remaining installment payments the loan, whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the the origination amortization term and the balance due term, are statistically significant at the .001 level. The percent of variance explained by the overall regression (the R^2) is only 5.41%; however, again as discussed above, the default incidence in the office loans was about 6.83% over the analysis period ending in 2021.

Table 6 presents the results of the logistic regression model for the multifamily loans in the upper panel of the table. Exactly as with the prior results, the largest positive and statistically significant effects (with χ^2 tests that reject the null at $< .0001$) on the default outcome are the utility cost as a percentage of expenses on the last observation month, the origination interest rate on the loan, and the indicator variable for the whether the loan was structured as an interest-only loan. Again, all of these covariates are associated with the willingness or ability to cover the remaining installment payments or the remaining balance which is the present value of these payments on the loan. The higher these costs, the greater the likelihood there will be a default. The next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the origination balance and the origination amortization term, are statistically significant at the .001 level. The likelihood ratio test indicates that the null of zero value coefficients is rejected at $< .0001$.

The results for the retail loans are presented in the middle panel of Table 4. Like the linear regression model, again the three strongest positive and statistically significant effects on the probability of default are the utility cost as a percentage of expenses on the last observation month, the interest rate on the loan, and the indicator variable for whether the loan was structured as an interest-only loan. As above, all of these covariates are associated with the willingness or ability to pay the remaining installment payments on the loan, whereby the higher these costs the greater the likelihood there will be a default. Again, the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the original loan balance, the origination amortization term, and the balance due term are significant at the .001 level. The likelihood ratio test indicates that the null of zero value coefficients is rejected at $< .0001$.

Finally, the results for the office loans are presented in the lower panel of Table 5. Although the utility cost as percentage of expenses has a positive coefficient there is more noise in the estimate for office loans, and it is only statistically significant at a level greater than 0.10. Similar to the results for multifamily and retail loans, the interest rate on the loan and the indicator variable for whether the loan was structured as an interest only loan both have statistically significant positive effects on default. As above, all of these covariates are associated with the willingness or ability to cover the remaining installment payments of the loan, whereby the higher these costs the greater the likelihood there will be a default. Again,

Table 5: **Linear probability estimates of default for multifamily, retail, and office mortgages where utilities are measured as percentage of total expenses.** This table presents the coefficient estimates for a linear regression of the probability of default (measured as a zero or one indicator variable) based upon samples of commercial mortgages by property type that are evaluated at the last observable month of performance for current loans and on the last recorded performance month for defaulted loans. We control for the origination characteristics of the mortgage, the age of the loan on the last observation day, state fixed effects and two default related channels: i) the likelihood of negative equity (measured as the origination loan-to-value ratio); ii) a measure of energy-related cost effects on cash flow on the last observation day, the "double trigger," (measured as the total cost of utilities as a percentage of total expenses).

Multifamily	Coefficient Estimate	Standard Error
Intercept	-.076***	0.009
Utility Cost to Total Expenses (%)	0.075***	0.02
Age of loan (Months)	0.00017***	0.00002
Origination Balance ($ Millions)	.00003	0.00005
Origination Loan-to-value ratio	0.0005***	0.00008
Origination Mortgage Interest rate	0.0093***	0.0013
Origination Amortization Term (Months)	-0.00004***	0.000001
Origination Balance Due Term (Months)	-0.00013***	0.00003
Interest Only Loan Indicator	0.0139***	0.0018
State fixed effects	Yes	
Number of observations	18,832	
R^2	0.029	
Retail	Coefficient Estimate	Standard Error
Intercept	-.321***	0.045
Utility Cost to Total Expenses (%)	0.2482***	0.0067
Age of loan (Months)	0.00038***	0.000082
Origination Balance ($ Millions)	0.0013***	0.0003
Origination Loan-to-value ratio (%)	0.0019***	0.00039
Origination Mortgage Interest rate (%)	0.0203**	0.0061
Origination Amortization Term (Months)	0.00011**	0.00004
Origination Balance Due Term (Months)	-0.00009	0.00017
Interest Only Loan Indicator	0.0773***	0.0078
State fixed effects	Yes	
Number of Observations	5,128	
R^2	0.051	
Office	Coefficient Estimate	Standard Error
Intercept	-0.2182***	0.0422
Utility Cost to Total Expenses (%)	0.1034**	0.0527
Age of loan (Months)	0.00044***	0.00008
Origination Balance ($ Millions)	-0.0002	0.00013
Origination Loan-to-value ratio (%)	0.0018***	0.00039
Origination Mortgage Interest rate (%)	0.0154**	0.0059
Origination Amortization Term (Months)	-0.00002	0.000039
Origination Balance Due Term (Months)	-0.000022	0.000017
Interest Only Loan Indicator	0.0789***	0.00825
State fixed effects	Yes	
Number of Observations	4,264	
R^2	.0541	

$^{***}P < 0.01$, $^{**}P < 0.0$

Table 6: **Logistic regression estimates of the probability of default for multifamily, retail, and office mortgages where utilities are measured as percentage of total expenses.** This table presents the coefficient estimates for a logistic regression of the probability of default (measured as a zero or one indicator variable) based upon samples of commercial mortgages by property type that are evaluated at the last observable month of performance for current loans and on the last of recorded performance month for defaulted loans. We control for the origination characteristics of the mortgage, the age of the loan on the last observation day, state fixed effects and two default related channels: i) the likelihood of negative equity (measured as the origination loan-to-value ratio); ii) a measure of the ration of utility costs to total operating expenses on the last observation day, the "double trigger," (measured as the total cost of utilities as a percentage of total revenues).

Multifamily	Coefficient Estimate	Wald Error	χ^2	Prob. $> \chi^2$
Intercept	-13.9511	1.0269	1814.559	$< .0001$
Utilities as % Expenses	4.775	1.204	15.740	$< .0001$
Age of Loan (Months)	0.0117	0.1013	32.5100	$< .0001$
Origination Balance ($ Millions)	-0.0036	0.0064	0.2682	0.6045
Origination Loan-to-Value Ratio (%)	0.0667	0.0117	32.5100	$< .0001$
Origination Interest Rate (%)	0.7237	0.1031	51.4899	$< .0001$
Origination Amortization Term (Months)	-.00089	0.00067	1.7651	.1840
Origination Balance Due Term (Months)	-0.0145	0.00283	26.2905	$< .0001$
Interest Only Loan Indicator	1.1475	0.1616	50.4384	$< .0001$
State Fixed Effects	Yes			
Number of observations	18,832			
Likelihood Ratio = 525.1330	Df = 13		Prob. $\chi^2 < .0001$	
Retail	**Coefficient Estimate**	**Wald Error**	χ^2	Prob. $> \chi^2$
Intercept	-9.4784	0.8198	133.6618	$< .0001$
Utilities as % Expenses	3.4527	0.9436	13.3883	$< .0003$
Age of Loan (Months)	0.0070	0.0015	20.0653	$< .0001$
Origination Balance ($ Millions)	0.0140	0.0034	13.3883	0.0003
Origination Loan-to-Value Ratio (%)	0.0425	0.0077	30.3907	$< .0001$
Origination Interest Rate (%)	0.3242	0.0943	11.8248	.0006
Origination Amortization Term (Months)	.0013	0.0006	4.1435	.0418
Origination Balance Due Term (Months)	-0.0045	0.00314	2.0156	.1557
Interest Only Loan Indicator	1.1121	0.1196	86.4766	$< .0001$
State Fixed Effects	Yes			
Number of observations	5,128			
Likelihood Ratio = 295.1955	Df = 13		Prob. $\chi^2 < .0001$	
Office	**Coefficient Estimate**	**Wald Error**	χ^2	Prob. $> \chi^2$
Intercept	-10.2273	0.9764	109.7245	$< .0001$
Utilities as % Total Expenses	1.6021	0.8636	3.4420	$< .0636$
Last Observation Age of Loan (Months)	0.0111	0.0021	30.0251	$< .0001$
Origination Balance ($ Millions)	-0.00322	0.00316	4.0100	0.0452
Origination Loan-to-Value Ratio (%)	0.0504	0.0095	28.3009	$< .0001$
Origination Interest Rate (%)	0.3340	0.1067	9.7906	.0018
Origination Amortization Term (Months)	-.00041	0.00063	.4310	.5115
Origination Balance Due Term (Months)	-0.00396	0.00317	1.5681	.2105
Interest Only Loan Indicator	1.3862	0.1554	79.6038	$< .0001$
State Fixed Effects	Yes			
Number of observations	4,264			
Likelihood Ratio = 287.561	Df = 13		Prob. $\chi^2 < .0001$	

the next largest and statistically positive effect was the proxy for the likelihood of negative equity as measured by the origination loan-to-value ratio. All of the other covariates have the expected sign and all but the original loan balance, the origination amortization term, and the balance due term, are significant at the .001 level. The likelihood ratio test indicates that the null of zero value coefficients is rejected at $< .0001$.

6 Cost Elasticity Estimates

The wholesale energy markets involves large scale participants such as utility companies, independent power producers, natural gas producers, and merchant traders. Prices in these markets ultimately determine price levels experienced by small scale consumers with energy loads being serviced by either utility companies or energy service providers. Figure 1 shows the time series of energy wholesale prices for natural gas and electricity at their respective most liquid trading hubs. One plot corresponds to monthly averages of daily prices of nearest maturity futures contracts traded at the New York Mercantile Exchange (NYMEX) for Henry Hub from January 2012 to February 2022. This graph also shows the plot of a 12-month rolling average price, which better reflects changes of price levels experienced by rate payers over time. Both plots show clearly that natural gas price levels are extremely volatile. In January of 2013, for instance, the 12-month rolling average price was $2.82/MMBtu. In less than 2 years, in November of 2014 prices went up to $4.34/MMBtu, corresponding to a 54% increase. In June of 2016 prices went down to $2.30/MMBtu. Again, in less than 2 years, a 47% decrease from its peak in 2014. More recently, natural gas market prices showed a pronounced upward trend, with the 12-month rolling average changing from $2.05/MMBtu in September of 2020 to $3.98/MMBtu in February of 2022, or a 94 % increase in just 18 months.

Figure 2 corresponds to monthly averages of daily on-peak day-ahead prices for the Pennsylvania, New Jersey, Maryland (PJM) Regional Transmission Organization (RTO) western Hub from January 2006 to February 2022. Similarly to the natural gas case, these are extremely volatile prices. More recently, for instance, the time series for 12-month rolling average recorded a price change from $24.31 to $49.67 from November 2020 to February 2022. This amounts to a 94% price increase over a period of only 16 months. This is equivalent to annualized rate of appreciation of 50%.

These results demonstrate how the volatility of energy prices can affect the financial performance of a commercial building during its mortgage term. Given that most utility expenses stems from the consumption of natural gas and electricity, a 30% change, say, in these expenses, constitute a common event over a one year term.

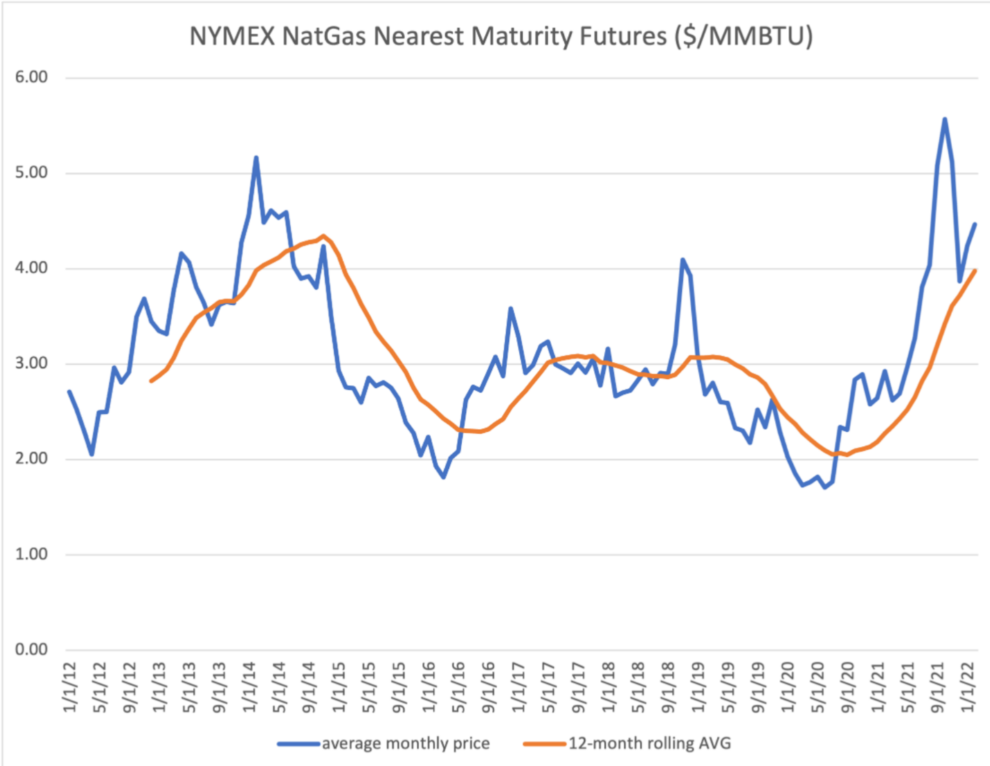

Figure 1: Monthly and 12-month rolling average natural gas prices for the nearest maturity NYMEX Henry Hub futures contracts.

	Utility Costs as % of Total expenses				
	Average	10% Shock in Energy Prices	Difference	30% Shock in Energy Prices	Difference
Multifamily	17.0%	18.4%	1.4%	21.0%	4.0%
Retail	14.9%	16.1%	1.2%	18.5%	3.6%
Office	22.0%	23.7%	1.7%	26.8%	4.8%

			Increase in Default Rate (bps)	
	Regression Coefficient	Average Default Rate in Basis Points (bps)	10% Shock in Energy Prices	30% Shock in Energy Prices
Multifamily	0.0750	129	10	30
Retail	0.2482	772	31	90
Office	0.1034	682	17	50

Table 7: Utility Cost Elasticity of Default

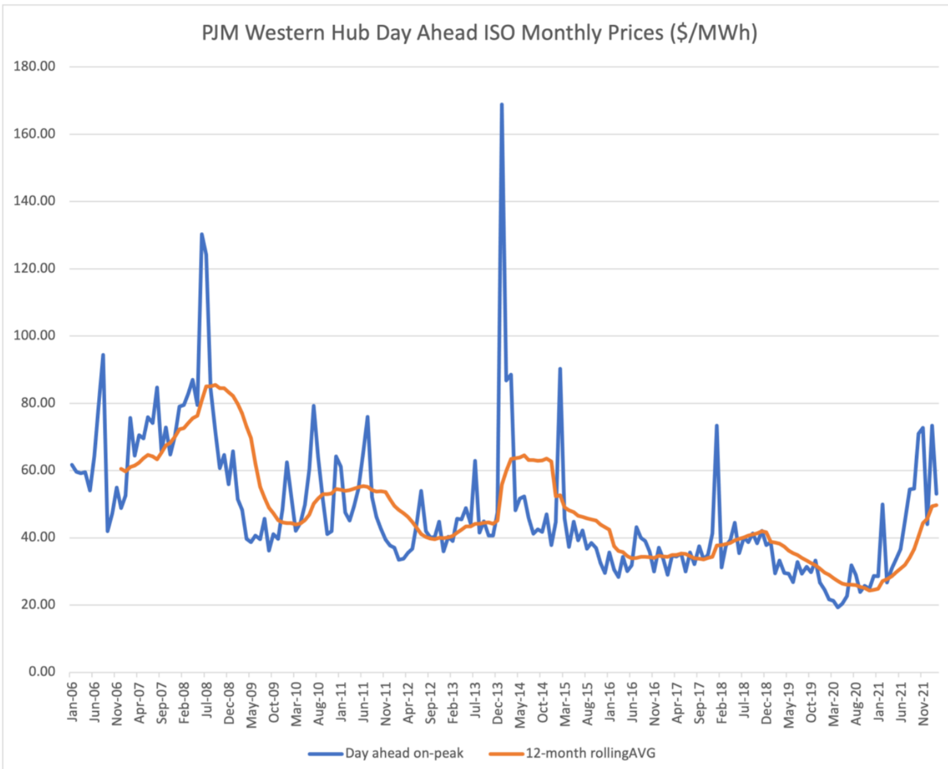

Figure 2: Monthly and 12-month rolling average day ahead prices for the PJM Western Hub electricity node.

From the regression coefficient estimates that were reported in Table 5 and the effects of historically likely 10% and 30% shocks to utility expenses caused by changes in energy prices as shown in Figure 1 and Figure 2, we are able to compute the associated shocks to the probability of commercial mortgage default. For example, as shown in Table 7, for multifamily buildings the average utility cost is 17.0% of total expenses. The regression coefficient for this building type is 0.0750. For a 30% shock on energy prices, the utility expenses increase to $(0.17 \times 1.3)/(1 + 0.17 \times 0.3) = 0.21$, or 21.0%, of total costs. This corresponds to a change of $21.0\% - 17.0\% = 4.0\%$. Thus, this shock in utility expenses translates into an increase in the probability of default of $0.0750 \times 4.0\% = 0.30\%$, or 30 basis points. The same approach is used to compute the increases in the probability of default to the other property types, as well as for a smaller 10% shock on the average utility costs.

Thus, we find that an historically likely 30% shock to energy prices leads to a 30 basis points increase in multifamily mortgage default, a 90 basis points increase in retail mortgage default, and a 50 basis points increase in office mortgage default. These estimates indicate that our overall results are not just statistically significant but also that the ratio of utility costs to total expenses has an economically relevant effect on the incidence of commercial mortgage default.

7 Conclusions

This report presents the results for a study of mortgage default for a large sample of securitized multifamily, retail, and office loans in the U.S. that were originated between 2002 and 2021. We report estimates for both linear probability and logistic regression models of mortgage default. Our empirical specification accounts both for traditional option exercise factors, such as the likelihood of negative equity, proxied by the origination loan-to-value ratio as well as co-incident factors, double triggers that are associated with the percentage of utility costs to total revenue on the last observation month of the mortgage. We find that both the option exercise and the double trigger channels are importantly associated with elevated default levels in these loans. These results provide a novel extension to the current literature on "double trigger" controls for default, by showing that the effect of shocks to energy factors on total revenue is directly related to the mortgage default.

Our reported results are based on a more direct measure of energy cost and efficiency than other recent studies focused on the association between building energy use and mortgage default (see, Issler et al., 2017, 2020a,b, 2021). These previous studies were hampered by small sample sizes and data challenges associated with dynamic building-specific measures of energy costs as well as other information on the on-going financial statements of the buildings. Interestingly, however, the new results of this larger national study with more than twenty eight thousand loans largely confirms prior findings that buildings with higher standardized energy usage are more vulnerable to default.

We find that the elasticity of default with respect to shocks to utility costs is both economically and statistically significant. Given the historical volatility of energy prices in the U.S., energy shocks of 30% or higher are probable. Such shocks would lead to nontrivial impacts on the probability of mortgage default for multifamily, office and retail properties.

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